HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1987

數學（課程甲） 試卷一
MATHEMATICS (SYLLABUS A) PAPER I

8.30 am–10.30 am (2 hours)

This paper must be answered in English

Attempt **ALL** questions in Section A and any **FIVE** questions in Section B.

Full marks will not be given unless the method of solution is shown.

87-CE-MATHS (SYL A) I–1

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area $\times$ height
PYRAMID	Volume	=	$\frac{1}{3}$ $\times$ base area $\times$ height

SECTION A **Answer ALL questions in this section.**
There is no need to start each question on a fresh page.
Geometry theorems need not be quoted when used.

1. Factorize

(a) $x^2 - 2x + 1$,

(b) $x^2 - 2x + 1 - 4y^2$.

(5 marks)

2. Find the values of a and b if $2x^3 + ax^2 + bx - 2$ is divisible by $x - 2$ and $x + 1$.

(5 marks)

3. Simplify

(a) $\sqrt{\dfrac{3^{5k+2}}{27^k}}$,

(b) $\dfrac{\log a^3 b^2 - \log ab^2}{\log \sqrt{a}}$.

(5 marks)

4. Solve the equation $\sin^2\theta = \frac{3}{2}\cos\theta$, where $0° \leqslant \theta < 360°$.

(6 marks)

5. α and β are the roots of the quadratic equation

$$kx^2 - 4x + 2k = 0 ,$$

where k $(k \neq 0)$ is a constant. Express the following in terms of k :

(a) $\alpha^2 + \beta^2$,

(b) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$.

(6 marks)

6.

Figure 1

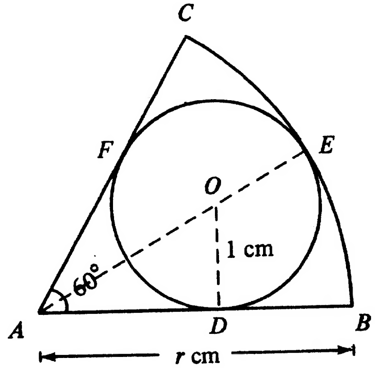
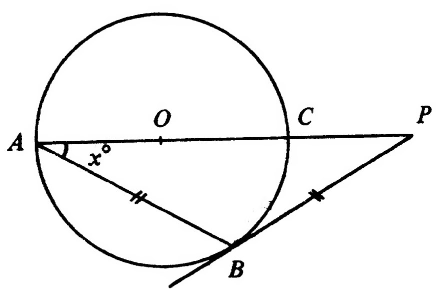


Figure 1 shows a circle, centre O, inscribed in a sector ABC. D, E and F are points of contact. $OD = 1$ cm, $AB = r$ cm and $\angle BAC = 60^\circ$. Find r.

(6 marks)

7.

Figure 2



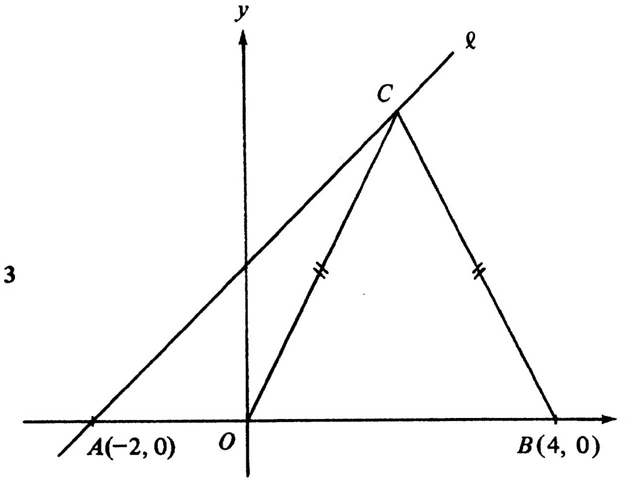
In Figure 2, O is the centre of the circle. $AOCP$ is a straight line, PB touches the circle at B, $BA = BP$ and $\angle PAB = x^\circ$. Find x.

(6 marks)

SECTION B Answer any FIVE questions from this section. Each question carries 12 marks.

8.

Figure 3



In Figure 3, O is the origin. A and B are the points $(-2, 0)$ and $(4, 0)$ respectively. ℓ is a straight line through A with slope 1. C is a point on ℓ such that $CO = CB$.

(a) Find the equation of ℓ.

(2 marks)

(b) Find the coordinates of C.

(3 marks)

(c) Find the equation of the circle passing through O, B and C.

(4 marks)

(d) If the circle OBC cuts ℓ again at D, find the coordinates of D.

(3 marks)

9.

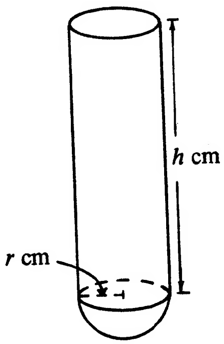


Figure 4a

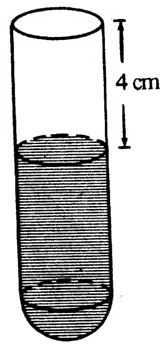


Figure 4b

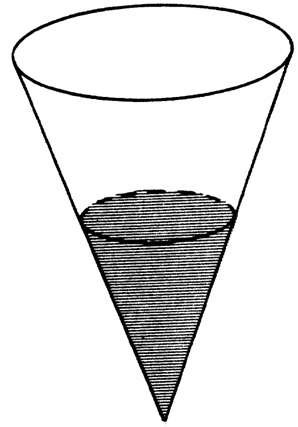


Figure 4c

Figure 4a shows a test-tube consisting of a hollow cylindrical tube joined to a hemispherical bowl of the same radius. The height of the cylindrical tube is h cm and its radius is r cm. The capacity of the test-tube is $108\pi\ \text{cm}^3$. The capacity of the hemispherical part is $\frac{1}{6}$ of the whole test-tube.

(a) (i) Find r and h.

(ii) The test-tube is placed upright and water is poured into it until the water level is 4 cm beneath the rim as shown in Figure 4b. Find the volume of the water. (Leave your answer in terms of π.)

(9 marks)

(b) The water in the test-tube is poured into a right circular conical vessel placed upright as shown in Figure 4c. If the depth of water is half the height of the vessel, find the capacity of the vessel. (Leave your answer in terms of π.)

(3 marks)

10.

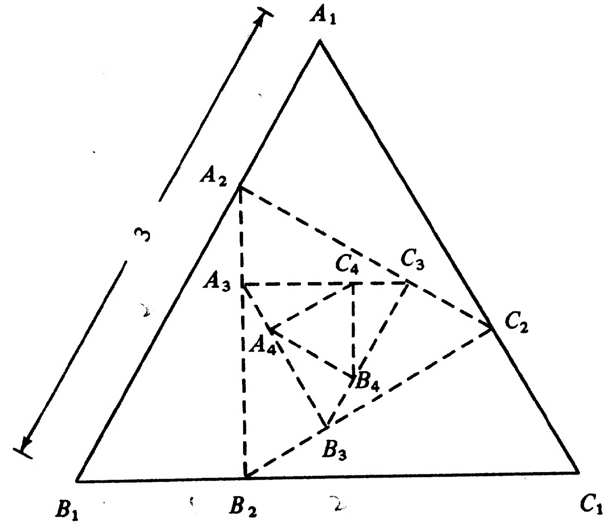


Figure 5

In this question you should leave your answers in surd form.

In Figure 5, $A_1B_1C_1$ is an equilateral triangle of side 3 and area T_1.

(a) Find T_1. (2 marks)

(b) The points A_2, B_2 and C_2 divide internally the line segments A_1B_1, B_1C_1 and C_1A_1 respectively in the same ratio 1 : 2. The area of $\Delta A_2B_2C_2$ is T_2.

(i) Find A_2B_2.

(ii) Find T_2. (4 marks)

(c) Triangles $A_3B_3C_3$, $A_4B_4C_4$, ... are constructed in a similar way. Their areas are T_3, T_4, ..., respectively. It is known that T_1, T_2, T_3, T_4, ... form a G.P.

(i) Find the common ratio.

(ii) Find T_n.

(iii) Find the value of $T_1 + T_2 + \ldots + T_n$.

(iv) Find the sum to infinity of the G.P. (6 marks)

11.

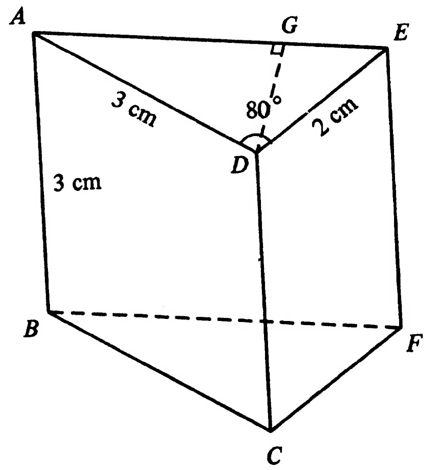


Figure 6

In this question, you should give your answers in cm or degrees, correct to 3 decimal places.

Figure 6 shows a solid in which $ABCD$, $DCFE$ and $ABFE$ are rectangles. DG is the perpendicular from D to AE. $AB = 3$ cm, $AD = 3$ cm and $DE = 2$ cm. $\angle ADE = 80°$.

(a) Find AE. (3 marks)

(b) Find $\angle DAE$. (3 marks)

(c) Find DG. (2 marks)

(d) Find BD. (2 marks)

(e) Find the angle between the line BD and the face $ABFE$. (2 marks)

12. **If you attempt this question, you should refer to the separate supplementary leaflet provided.**

A factory produces three products A, B and C from two materials M and N.

Each tonne of M produces 4000 pieces of A, 20 000 pieces of B and 6000 pieces of C.

Each tonne of N produces 6000 pieces of A, 5000 pieces of B and 3000 pieces of C.

The factory has received an order for 24 000 pieces of A, 60 000 pieces of B and 24 000 pieces of C. The costs of M and N are respectively \$4000 and \$3000 per tonne. By following the steps below, determine the least cost of the materials used so as to meet the order.

(a) Suppose x tonnes of M and y tonnes of N were used. By considering the requirement of A, B and C of the order, five constraints could be obtained. Three of them are:

$$x \geqslant 0\,,$$
$$y \geqslant 0\,,$$
$$4000x + 6000y \geqslant 24\,000\,.$$

Write down the other two constraints on x and y. (2 marks)

(b) On the graph paper provided, draw and shade the region which satisfies the five constraints in (a). (6 marks)

(c) Express the cost of materials in terms of x and y.

Hence use the graph in (b) to find the least cost of materials used to meet the order. (4 marks)

13. P, Q and R are three bags. P contains 1 black ball, 2 green balls and 3 white balls; Q contains 4 green balls; R contains 5 white balls. A ball is drawn at random from P and is put into Q; then a ball is drawn at random from Q and is put into R. Find the probability that

(a) the black ball still remains in P, (2 marks)

(b) the black ball is in Q, (4 marks)

(c) the black ball is in R, (3 marks)

(d) all the balls in R are white. (3 marks)

14. **If you attempt this question, you should refer to the separate supplementary leaflet provided.**

Figure 7 shows the graph of $y = x^3 - 6x^2 + 9x$.

(a) By adding suitable straight lines to the figure, find, correct to 1 decimal place, the real roots of the following equations:

(i) $x^3 - 6x^2 + 9x - 1 = 0$,

(ii) $x^3 - 6x^2 + 10x - 6 = 0$. (6 marks)

(b) By using the method of magnification, find, correct to 2 decimal places, the real root(s) of (a)(ii). (3 marks)

(c) From the graph in Figure 7, find the range of values of k such that the equation $x^3 - 6x^2 + 9x - k = 0$ has three distinct real roots. (3 marks)

END OF PAPER

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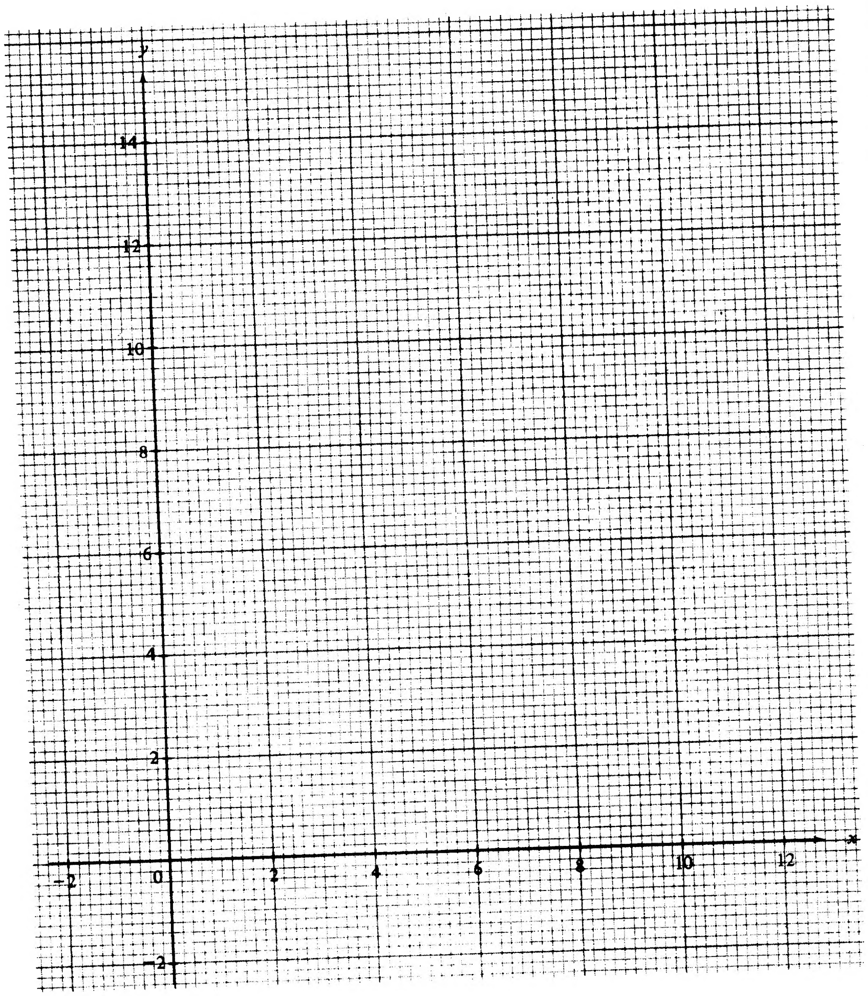
數學(課程甲) 試卷一(附頁)
MATHEMATICS (SYLLABUS A) PAPER I
(SUPPLEMENTARY LEAFLET)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

12. **If you attempt this question, fill in the details in the first three boxes above and tie this sheet inside your answer book.**



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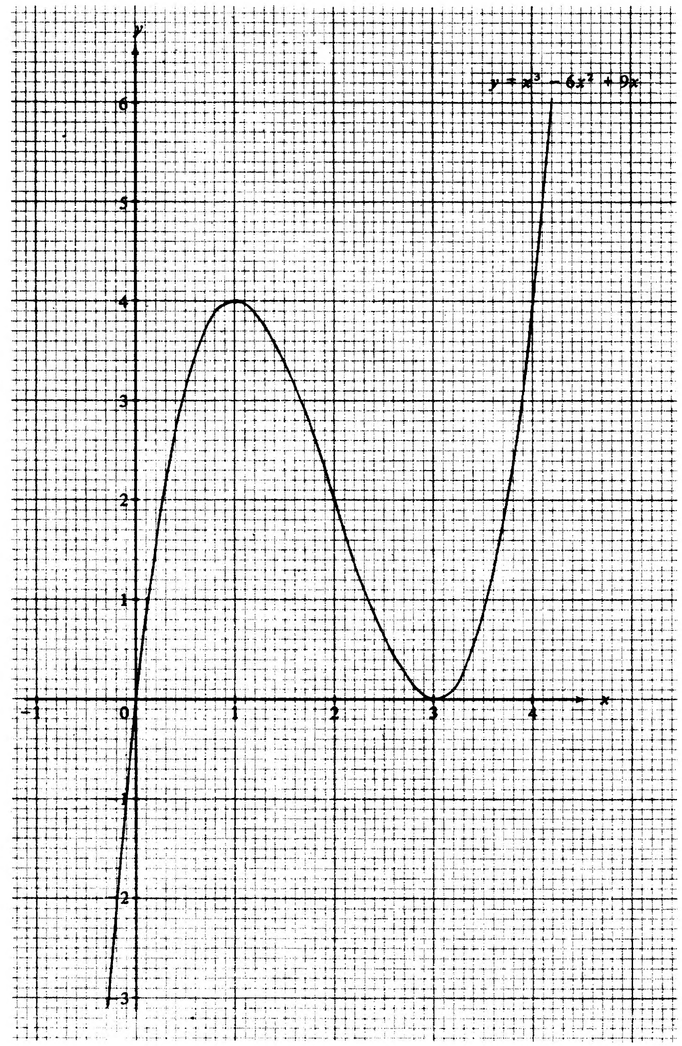

Figure 7